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ABSTRACT

This article describes constructivist techniques for teaching concepts of decimals and metrics through the combination of innovative measuring tools, activities and videos in classrooms. Even though the chronological age of the students involved differed--fifth and sixth graders or pre-service teachers--it was discovered that similar activities helped all students learn the decimal and metric systems. Through teacher observation and informal interviews made by the authors, recommendations are made to upper-elementary math and science teachers as well as mathematics and science methods teachers at the university level. (Author/WRM)

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Running head: Deci-mania!

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Deci-mania!

Teaching Teachers *and* Students

Conceptual Understanding of Our Decimal System

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Abstract

This article describes constructivist techniques for teaching concepts of decimals and metrics through the combination of innovative measuring tools, activities and videos in classrooms. Even though the chronological age of students differs, fifth and sixth graders or pre-service teachers, it was discovered that similar activities helped all students learn the decimal and metric systems. Through teacher observation and informal interviews made by the authors, recommendations are made to upper-elementary math and science teachers as well as mathematics and science methods teachers at the university level.

Deci-mania! Teaching Teachers *and* Students

Conceptual Understanding of Our Decimal System

When it comes to effectively using the decimal system to manipulate science equipment and data, teachers find that many students seem to fall short. "What are these kids learning in math *anyway*?" Such lamentations come from the lips of science teachers everywhere. Many science teachers spend several days of class time reteaching decimal concepts just so they can finish units that rely heavily on accuracy in measurement and understanding of numerical values. Why is that?

It's not that decimals aren't being taught --students visit the decimal system many times throughout their elementary and middle school years. The problem lies in that the emphasis on place value and decimals tends to be on computation, not conceptual understanding. For students to be successful in using decimals, they must first understand them. "When children possess a sound understanding of fraction and decimal concepts, they can use this knowledge to describe real-world phenomena and apply it to problems involving measurement, probability, and statistics" (National Council of Teachers of Mathematics (NCTM), 1989, p 57.). Thus, science teachers can spend more time teaching science!

Teachers can help students gain conceptual understanding of decimals by making them "real" because "*knowing* mathematics is *doing* mathematics" (NCTM, 1989, p. 7). Simply "visiting" decimals year after year won't do; students must experience decimals, or rather, *decimal fractions*. Tracy (1994) suggests that we use proper mathematical language to enhance the conceptual understanding of our students. Students need to hear teachers say "decimal fraction" rather than just hearing "decimal." Certainly there are students who, when hearing "decimal," focus their attention on the dot or the thing that looks like a period. We suggest that teachers help students focus on the "many-ness" of a decimal fraction rather than

just the dot. Moreover, this mathematical language connects decimal fractions to common fractions as advocated in the Curriculum and Evaluation Standards (1989).

What do decimal fractions look like? How big are they? How little are they? Who needs 'em and what is that decimal point anyway? These types of questions can be answered by students themselves when provided with plenty of cognitively appropriate experiences.

The Right Stuff

With combined experience of 29 years of teaching (the first author teaches upper elementary and the second author currently teaches math methods at the university level), we have found a combination of excellent teaching materials and activities that provide elementary and middle school students, as well as future and practicing teachers, with the educational encounters they need to make sense of decimal fractions. Through connecting carefully designed activities including: (a) using metric measuring tools, (b) making a Mega-Number Line, (c) playing with Decimal Squares®, (d) viewing thought-provoking videos, (e) using the Math Explorer® or Math Explorer Plus® calculator, and (f) linking to the United States Metric Association website, students' and teachers' understanding of decimal fractions can be greatly enhanced.

Metric measurement is a natural way for students to experience the base ten system in a concrete learning mode. Measurement equipment lends itself to hands-on learning and sense making. Metric tools such as meter sticks and measuring tapes are commonly found items in schools. These help students measure objects in centimeters, decimeters, and meters. Trundle wheels assist in measuring longer distances such as dekameters, hectometers, and even kilometers. Calipers are nifty gadgets utilized to measure precise distances, such as the width of a nose, the thickness of a pencil, and the diameter of the inside of a milk jug. The Square Meter

Overlay® is fantastic for measuring square units, the key to measuring area.

The Mega-Number Line is another activity used to enhance conceptual understanding. Students measure out ten meters of adding machine tape. They then construct a number line from zero to one, marking the tenths, hundredths, and thousandths. Students discover as they work on this cooperative activity that one-tenth of ten meters is one meter, one one-hundredth of ten meters is one decimeter, and so on.

Decimal Squares® is an excellent program for developing conceptual understanding of decimal fractions in a pictorial learning mode. It consists of two sets of color-coded cards (green for tenths, red for hundredths, and yellow for thousandths). The first set of cards shows pictorial representations of decimal fractions, and the second set shows the corresponding numerical symbol and written word form of the decimal fraction. A teacher's guide and accompanying activities are also included. Decimal Squares® correlates nicely with traditional base ten blocks, and offers an excellent bridge between the concreteness of the manipulatives and the abstractness of the numerical representation of decimal fractions.

Videos are seldom used as resources in mathematics. But two videos, The Powers of Ten, produced by the Canada Film Board, and Sunburst's Patterns with Ten are essential tools to help students see the relevance of and marvel at the base ten system. While similar in concept, each has a distinct perspective and purpose. Together they offer a holistic view of decimal fractions.

Students Give it the "Thumbs Up"

Metric Measuring Tools

Many students and teachers have practiced metric measurement using rulers, meter sticks and metric measuring tapes. While these tools are a necessary and

invaluable component of the learning experience, we find them to be rather mundane. Trundle wheels, less ubiquitous and more costly ($25), are often a new adventure for students --both young and old*er*. The physical activity, the clicking sound, the challenge of keeping track of the click, and the granting of every student's wish to "go outside today" all add up to a multimedia measuring experience that is nearly unsurpassed. That is, until they meet the caliper.

If meter sticks are mundane, calipers must be captivating! Capable of measuring to the hundred-thousandth of a meter, calipers are quite outside anything our students have experienced. Since they can be found in school catalogs for as little as $4.95 each, we believe that every upper elementary or middle school mathematics classroom should have a set. Whether students are aged 9, 19, 29, or 39, after they learn how to use this precision-tool they become mathematically empowered. Their ability to transfer mm to cm and mm to m becomes fine-tuned.

Imagine a huge overhead transparency measuring one square meter. Now draw the lines that make the 100 square decimeters within. Then draw the lines on one square decimeter that make the 100 square centimeters. Add a few labels and you would have a Square Meter Overlay®. The second author made a prototype for such a tool in about two hours on a large sheet of translucent plastic. Now they are available commercially. Both adults and middle school students are amazed at their eventual conceptual understanding of ideas like 2.38 m^2 = 238 dm^2 = 23,800 cm^2!

Believe it or not, when our students get their hands on a caliper, trundle wheel, or Square Meter Overlay® it seems as if they are compelled to go forth and measure! We have observed students' eyes rapidly darting from one object to another in search of the perfect thing to measure. We have been inundated with young*er* students yammering, "Canwego measure the halls, canwego around the school, canwego to the snack shop?" Often other classmates follow behind with

similar passion. For olde*r* students we have observed a rush of elation as they toil over their metric tools and announce, "Now I get it!" Inevitably, controversy erupts and soon students engage in meaningful discourse as they struggle to read the measuring tools with unequivocal accuracy. *These* are Worthwhile Tasks (NCTM, 1991).

Mega-Number Line

Connecting the metric system to the decimal system is no small task. We have a mega-solution: THE MEGA-NUMBER LINE. Simply speaking, this is a super-enlarged segment, from zero to one, of a traditional number line. The ten-meter segment of adding machine tape represents one whole. When divided into **ten equal pieces**, each piece becomes one-tenth of the whole; when each tenth is divided into **ten equal pieces**, each new segment becomes one-hundredth of the whole, and so on. This process pictorially represents the negative powers of ten and again gives students a"precise" purpose for using their metric tools.

The Mega-Number Line is a cooperative assignment that takes students out of their seats where they discover the relationships among the metric system and decimal fractions. Each student becomes an essential member of the team (manager, two expert measurers, and recorder or "scribe"), whose input is needed to create an accurate representation of the all the tenths, hundredths, and thousandths between zero and one. Problem-solving skills, *Standard 1* (NCTM, 1989), are applied as groups plan their strategies for marking the segments. W ith each successfully completed power of ten, groups earn their choice of bingo-style colored stamps from the teacher. Students who have a good understanding of number line theory quickly realize that the increments are compatible with their metric measuring tools. Students who do not have this understanding receive small-group instruction from the teacher. Loads of informal assessment (NCTM, 1995) by the teacher enhances the

learning of all students. Creating the number line provides *rich opportunities* (Mathematical Sciences Education Board (MSBE) & National Research Council (NRC), 1993, p. 6) where kids *and* teachers are really Measuring Up! (MSBE & NRC, 1993).

Common misconceptions about decimal fractions surface immediately as students attempt to draw hash marks and label them correctly. A majority of students do not understand that 0.1 (one-tenth) is the same as 0.10 (ten hundredths) which is preceded by 0.09 (nine hundredths) and is followed by 0.11 (eleven hundredths). This roadblock is solved by making students count the hundredths out loud, while moving their fingers along the hash marks. Most students remember being told that they can "put zeros after" a decimal fraction without changing its value. However, most students don't understand what that really means. The Mega-Number Line activity enables them to finally make sense of that well-known rule. Once this concept is in place (all puns intended), labeling the thousandths becomes an easy, but lengthy task.

The Mega-Number Line activity is a highly motivating project that teaches a myriad of decimal concepts in a fun way. When students are finished with the number line, they proudly display them on the walls of the school. The culminating question students must answer is "How many numbers are there between zero and one?" When the groups consider that they theoretically could break each hash mark into ten equal pieces over and over again, they come to the awesome realization that there are infinite negative powers of ten between EVERY hash mark ever drawn or imagined ...

Decimal Squares®

Decimal Squares® takes the idea of the Mega-Number Line and places it in the format of area. These plastic coated, color-coded squares begin as one whole.

When divided into **ten equal pieces**, each red picture shows the tenths. When each tenth is divided into **ten equal pieces**, each green picture shows the hundredths, and so on. This consistency matches the pattern of the Mega-Number Line-activity, where students are dividing something into ten equal pieces, providing multiple perceptions (Dienes, 1963) of the same mathematical concept. Decimal Squares® also bridge the concrete experience of the Mega-Number Line to the abstract experience of the math book.

When students first see Decimal Squares®, their curiosity compels them to ask what they are and when do they get to use them. The mere resemblance to a deck of cards is enough for students to decide that whatever they are, they must be fun. When introduced to the pictorial cards, the first response from teachers and students is always, "cool!" Those who can read the picture cards fluently enjoy the feeling of success when matching them with the corresponding cards that have the written word form and the symbolic number form printed on them.

The Decimal Squares® program includes many activities to teach operations with decimal fractions. We designed an activity (see Figure 1) that connects the decimal fractions with their corresponding common fractions and also bridges students' pictorial and symbolic understanding. In pairs, students pick five symbolic number cards from the deck and place them in order from least to greatest. Students must locate the corresponding picture card to confirm that 0.25, 0.3, and 0.325, for example, are in proper sequence. Then they write each decimal fraction as a common fraction with 1000 in the denominator. Next, they use the Math Explorer® calculator (F ◄► D) key to convert the common fraction into a decimal fraction, and back again to a common fraction. Finally, they write the word form of the last fraction shown, in common fraction form, on the calculator.

Sometimes after students use the (F ◄► D) key with fractions such as 0.125, the common fraction doesn't change (125/1000), and neither does the decimal fraction. But in other cases, such as 0.500 (five-hundred thousandths), when the common fraction (500/1000) is transformed into a decimal fraction, the calculator automatically truncates or converts it into the lowest terms (0.5). When the decimal fraction is converted back to a common fraction, it is no longer 500/1000, but 5/10! Students then write in their math journals about why they think the symbols changed, yet the number values stayed the same. Again, teachers may apply alternative assessment (Ann Arbor, 1993) strategies as students describe in their own terms the concepts of equivalency and simplifying fractions.

Videos

At any time during decimal fraction instruction students must watch two videos that demonstrate the power and usefulness of fractions. The Powers of Ten, (1978) produced by the Canada Film Board, takes viewers on a fantastic journey (nine mins long) as far away as 10^{24} meters (100 million light years) from the Earth and as close as 10^{-16} meters (0.000001 ångstroms) into a person's body. Prior to beginning the journey we tell our students, "Put your seat belts on!" Starting only one meter from a man sleeping on a blanket, the camera moves away at graduated powers of ten. Travelers soon see Soldiers' Field, then Chicago, then Lake Michigan, then the Mid-west, then the United States and Canada, then the Earth, then the orbit of our moon... until we've reached "the limit of our vision" (Canada Film Board, 1978), far out into the universe. The camera then moves back toward the man at a faster rate. When it reaches its starting point, it continues to move closer at graduated negative powers of ten, until it has reached a quark, currently, the smallest known particle within a cell.

For more "earthly" examples of tens, Sunburst's Patterns With Ten (1993)

highlights more commonly understood examples in society. One million pennies and one kiloliter of soda pop makes for a memorable visual experience. This video, only 11 minutes long, emphasizes the ease and usefulness of the metric system in science and industry. We recommend that you show both of these videos more than once if you have the time.

For Teachers Only

There are just two more opportunities of which every excellent teacher of math should take advantage. The United States Metric Association website is dedicated to educating all citizens of the nation about metirc history, current metric laws, educational materials, and other companies that have gone 100% metric. The website is listed in the Resource List.

Last but not least, the National Institute of Standards and Technology, a governmental agency, offers a FREE classroom starter kit to all teachers. It includes posters, a brief history of metrics in the United States, 35 metric rulers and 35 conversion charts.

Metric measurement is fraught with experiences to engage students and teachers in decimal fraction activities.

Reflections

If it is not obvious by now, we believe this collection of curricular activities and materials is *both* necessary and cognitively appropriate for *both* students and teachers. Most likely, pre-service and in-service teachers learned much of their mathematics through persistence of rule memorization and sheer drill and practice. And since the NCTM Professional Standards (1991) point out that teachers' experiences "...while learning mathematics have a powerful impact on the education they provide their students" (p. 127), it is incumbent upon all teachers of math to gain their own conceptual understanding of decimal fractions. Admittedly,

both authors *used to* teach mathematics the way they were taught; but, with the guidance of all three NCTM Standards documents and the positive reinforcement from our students, we have truly become better teachers and learners of mathematics. In the end, the upper elementary and middle school students are the beneficiaries of our combined efforts.

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Nebraska Mathematics and Science Initiative. (1993). Patterns with ten [Educational Video]. (Available from Sunburst Communication, 101 Castleton Street, P.O. Box 100, Pleasantville, NY 10570-0100, 1- 800-321-7511)

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Resource List

Calipers from Cuisenaire Company of America, P.O. Box 5026, White Plains, NY 10602-5026, 1-800-237-0338

Decimal Squares® from Nasco, 901 Janesville Avenue, Fort Atkinson, WI 53538-0901, 1-800-558-9595

FREE Starter Kits for Teachers from National Institute of Standards and Technology, Building 820, Room 306, Gaithersburg, MD 20899

Square Meter Overlay® from (S)he Learning Unlimited, 3774 N. Shimmons Circle, Auburn Hills, MI 48326, 1-248-370-0929

Trundle Wheels from Delta Education, P.O. Box 3000, Nashua, NH 03061-300, 1-800-442-5444

United States Metric Association Website http://lamar.colostate.edu/~hillger

Name That Decimal Square®!

Directions: Use 5 cards from the symbolic deck and your Math Explorer® calculator to complete this chart.

I. Line 'em up!	II. Convert 'em!	III. Math Explore® 'em!
* put in order from least to greatest		enter f → d — f → d — word form
0.125	$\frac{125}{1000}$	0.125 → $\frac{125}{1000}$ → one hundred twenty-five thousandths
0.15	$\frac{150}{1000}$	0.15 → $\frac{15}{100}$ → fifteen hundredths
0.		→ →
0.		→ →
		→ →
0.		→ →